MG26018 Simulation Modeling and Analysis

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Fall 2019

Assignment 2

Due Date: November 7 (in class)

Instruction

(a) You can answer in English or Chinese or both.

(b) Show enough intermediate steps.

(c) Write the answers independently.

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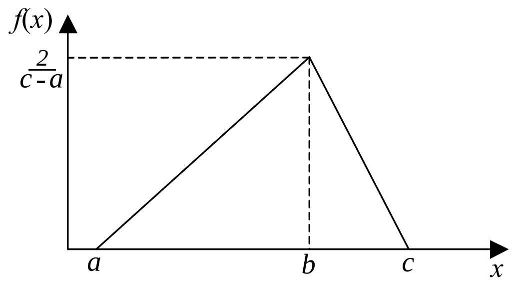
Question 1 ($10 + 15 = 25$ points)

The density of a triangular distribution is defined by three critical values: *min*, *max* and *mode* (see the following figure, where $min = a$, $mode = b$ and $max = c$).



(1) Given that we know how to generate random numbers, design a method to generate random variates from the triangular distribution with parameters a, b and c. (*Note:* Show the intermediate calculation.)

(2) Let $a = 0$, $b = 3$ and $c = 5$. Use whatever software/language (e.g., Excel, Matlab, C, Java, Python, etc.) to implement the method in (1) and generate 1000 random variates. Plot the histogram of the generated data. (*Note:* You don't have to present the data; just show the histogram. Don't use the built-in function of triangular distribution in the software/language. Describe the intermediate steps (for generating random variates) if you use software like Excel, or show the code (for generating random variates) if you use certain programming language.)

Question 2 (15 + 10 = 25 points)

To generate random variates from $\mathcal{N}(0,1)$, we can use either Cauchy(0) density or Laplace(0, 1) density as instrumental density. Cauchy(0) has density function $f(x) = \frac{1}{\pi(1+x^2)}$, $x \in (-\infty, \infty)$, and Laplace(0, 1) has density function $f(x) = \frac{1}{2}e^{-|x|}$, $x \in (-\infty, \infty)$.

(1) Show that for both Cauchy(0) and Laplace(0, 1), we are able to generate random variates from them using the inverse-transform technique. (*Notice that this is a prerequisite for them to be used as instrumental density.*)

(2) Which instrumental density gives us higher acceptance rate? Show the details.

Question 3 (10 points)

Recall the definition of sample variance: $S^2 = \frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}$, where $X_1, \ldots, X_n$ are a random sample of X, and $\bar{X}$ is the sample mean. Prove that S^2 is an unbiased estimator of the variance of X, i.e., $\mathbb{E}[S^2] = \text{Var}(X)$. (*This helps you better understand why the denominator is* $n-1$ *instead of* n.)

Question 4 (15 points)

Suppose $X_1, \ldots, X_n$ are an iid sample from Weibull(α, β) in shape & scale parametrization. The density function of Weibull(α, β) is $f(x) = \alpha\beta^{-\alpha}x^{\alpha-1}e^{-(x/\beta)^\alpha}$, $x > 0, \alpha > 0, \beta > 0$. Suppose the parameter α is known. Find the estimator of β using MLE.

Question 5 (10 + 15 = 25 points)

For the illustrative example on lecture note Lec4 pages 45-49, the first considered exponential distribution is rejected. We then consider the Weibull family (see its density function in **Question** 4). Suppose the parameters are estimated from the data via MLE: $\alpha = 0.525$, $\beta = 6.227$.

(1) Make the Q-Q plot. (*Note*: Show the necessary calculation. Use Excel or other software/language to draw the final plot.)

(2) Use K-S test to see if we would like to reject Weibull(0.525, 6.227) at level of significance $\alpha = 0.1,\ 0.05,\ 0.01$. (*Note*: You can use Excel or other software/language to compute the value of test statistic D; but implement the formula of D by yourself. The $(1-\alpha)$-quantile of D is $d_{n,1-\alpha} = c/\sqrt{n}$, and the value of c is given in the following table.)

	$1-\alpha$			
n	**0.900**	**0.950**	**0.975**	**0.990**
10	0.679	0.730	0.774	0.823
20	0.698	0.755	0.800	0.854
50	0.708	0.770	0.817	0.873
∞	0.715	0.780	0.827	0.886